Supplementary data for
"Binary self-dual codes with an automorphism of order 11"
by N. Yankov, M.H. Lee, M. Gürel, M. Nikolova

Binary self-dual $[66, 33, 12]$ codes with new values of the parameter

- $W_{66,2}$ for $\beta = 55$, gen $C_\varphi = \begin{pmatrix} e & 0 & 0 & 0 & \delta & \delta^{20} \\ 0 & e & 0 & \delta^2 & x^9\delta^{60} & x^9\delta^{10} \\ 0 & 0 & e & \delta^{40} & x^9\delta^{53} & x^9\delta^3 \end{pmatrix}$, support of $C_\pi = \{1,4\},\{2,5\},\{3,6\}$. Orthogonality check for C_φ with Hermitian inner product $(u,v) = \sum_{i=1}^{6} u_i v_i^{32}$:

$row1 * row1 = e + \delta^{33} + (\delta^{20})^{33} = e + \delta^{33} + \delta^{660} = e + \delta^{33} + \delta^9 = (x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^8 + x^9 + x^{10}) + (x + x^2 + x^3 + x^8 + x^9 + x^{10}) + (x^4 + x^5 + x^6 + x^7) = \mathbf{0}$
$row1 * row2 = \delta.(x^9\delta^{60})^{32} + \delta^{20}.(x^9\delta^{10})^{32} = x^{9.32}\delta^{1921} + x^{9.32}\delta^{340} = x^9(\delta^{61} + \delta^{61}) = \mathbf{0}$
$row1 * row3 = \delta.(x^9\delta^{53})^{32} + \delta^{20}.(x^9\delta^3)^{32} = x^9(\delta^{1697} + \delta^{116}) = x^9(\delta^{23} + \delta^{23}) = \mathbf{0}$
$row2 * row2 = e + (\delta^2)^{33} + (x^9\delta^{60})^{33} + (x^9\delta^{10})^{33} = e + \delta^3 + \delta^{27} + \delta^{15} = (x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^8 + x^9 + x^{10}) + (x^5 + x^6) + (x + x^2 + x^9 + x^{10}) + (x^3 + x^4 + x^7 + x^8) = \mathbf{0}$
$row2 * row3 = \delta^2(\delta^{40})^{32} + x^9\delta^{60}(x^9\delta^{53})^{32} + x^9\delta^{10}(x^9\delta^3)^{32} = \delta^{1282} + \delta^{1756} + \delta^{106} = \delta^{73} + \delta^{82} + \delta^{13} = (x^3 + x^4 + x^6 + x^{10}) + (x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^{10}) + (x + x^2 + x^5 + x^7) = \mathbf{0}$
$row3 * row3 = e + (\delta^{40})^{33} + (x^9\delta^{53})^{33} + (x^9\delta^3)^{33} = e + \delta^{1320} + \delta^{1749} + \delta^{99} = e + \delta^{18} + \delta^{75} + \delta^6 = (x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^8 + x^9 + x^{10}) + (x + x^3 + x^8 + x^{10}) + (x + x^2 + x^4 + x^5 + x^6 + x^7 + x^9 + x^{10}) + (x + x^{10}) = \mathbf{0}$.

$$gen\ C = \begin{pmatrix}
0111111111100000000000000000000000000000000101000110110100111111 \\
1011111111100000000000000000000000000000000110100011011010011111 \\
1101111111100000000000000000000000000000000111010001101101001111 \\
1110111111100000000000000000000000000000000111010001111101001111 \\
1111011111100000000000000000000000000000000101110100011111010011 \\
1111101111100000000000000000000000000000000110111010001111101001 \\
1111110111100000000000000000000000000000000110111010011111101001 \\
1111111011100000000000000000000000000000000110111010111111110100 \\
1111111101100000000000000000000000000000000110111010111111111010 \\
1111111110100000000000000000000000000000000100011011100011111101 \\
0000000000001111111111000000000000011011001010000110001011111011100 \\
0000000000001011111111100000000000011011001011000110001001111101110 \\
0000000000001101111111100000000000010110110010010001100010011111011 \\
0000000000001110111111100000000000010110110010100011000100111111011 \\
0000000000001111011111100000000000010101101100010100011001100111101 \\
0000000000001111101111100000000000010101101100010100011011100111110 \\
0000000000001111110111100000000000010101101100010100011011100111111 \\
0000000000001111111011100000000000010010101101100010100011011100111 \\
0000000000001111111101000000000000011001010110110001010001101110011 \\
0000000000000000000000000111111111011101011110000101100100011000000 \\
0000000000000000000000010111111111101110101111000010110000001100000 \\
0000000000000000000000110111111111011101011010000101100000110000 \\
0000000000000000000001110111111111101110101001000010110000011000 \\
0000000000000000000011110111111111011101101001000010100000001100 \\
0000000000000000000011111011111011101110110110010000100000000110 \\
0000000000000000000011111101111011101110011001000010000000011 \\
0000000000000000000011111110111011101111011001000010000000001 \\
0000000000000000000011111111011101011101101010011000100000000 \\
0000000000000000000011111111101101011101001011001000110000000 \\
1111111111100000000000000000000011111111111000000000000000000000 \\
0000000000011111111111000000000000000000000000000000011111111111 \\
0000000000000000000001111111111100000000000111111111110000000000
\end{pmatrix}$$

```
IsSelfDual(c);
AutomorphismGroup(c);
PartialWeightDistribution(c,18);
true
Permutation group acting on a set of cardinality 66
Order = 11
    (1, 6, 11, 5, 10, 4, 9, 3, 8, 2, 7)(12, 17, 22, 16, 21, 15, 20, 14, 19, 13,
        18)(23, 28, 33, 27, 32, 26, 31, 25, 30, 24, 29)(34, 39, 44, 38, 43, 37,
        42, 36, 41, 35, 40)(45, 50, 55, 49, 54, 48, 53, 47, 52, 46, 51)(56, 61,
        66, 60, 65, 59, 64, 58, 63, 57, 62)
[ <0, 1>, <12, 1298>, <14, 17358>, <16, 198561>, <18, 1602315> ]
    Running Magma V2.20-7.
    Seed: 2409036331; Total time: 3.240 seconds; Total memory usage: 219.06MB.
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- $W_{66,2}$ for $\beta = 88$, gen $C_\varphi = \begin{pmatrix} e & 0 & 0 & e & \delta^3 & \delta^{34} \\ 0 & e & 0 & \delta^{21} & x^8\delta^{80} & x^4\delta^{17} \\ 0 & 0 & e & \delta^{52} & x^7\delta^{85} & x^8\delta^{81} \end{pmatrix}$, support of $C_\pi = \{1,5\},\{2,6\},\{3,4\}$

Orthogonality check for C_φ with Hermitian inner product $(u,v) = \sum_{i=1}^{6} u_i v_i^{32}$:

$row1 * row1 = e + e + (\delta^3)^{33} + (\delta^{34})^{33} = \delta^6 + \delta^{1122} = \delta^6 + \delta^6 = \mathbf{0}$

$row1 * row2 = (\delta^{21})^{32} + \delta^3.(x^8\delta^{80})^{32} + \delta^{34}.(x^4\delta^{17})^{32} = \delta^{672} + x^{256}\delta^{2563} + x^{128}\delta^{578} = \delta^{21} + x^3\delta^{52} + x^7\delta^{20} = (x^2+x^3+x^4+x^5+x^6+x^7+x^8+x^9) + (1+x+x^7+x^9) + (1+x+x^2+x^3+x^4+x^5+x^6+x^8) = \mathbf{0}$

$row1 * row3 = e.(\delta^{52})^{32} + \delta^3.(x^7\delta^{85})^{32} + \delta^{34}.(x^8\delta^{81})^{32} = \delta^{1664} + x^4\delta^{2723} + x^3\delta^{2626} = \delta^{83} + x^4\delta^{26} + x^3\delta^{22} = (x^2+x^3+x^5+x^7) + (x^3+x^6+x^7+x^8) + (x^2+x^5+x^6+x^8) = \mathbf{0}$

$row2 * row2 = e + (\delta^{21})^{33} + (x^8\delta^{80})^{33} + (x^4\delta^{17})^{33} = e + \delta^{42} + \delta^{36} + \delta^3 = (x+x^2+x^3+x^4+x^5+x^6+x^7+x^8+x^9+x^{10}) + (x+x^3+x^4+x^5+x^6+x^7+x^8+x^{10}) + (x^2+x^5+x^6+x^9) + (x^5+x^6) = \mathbf{0}$

$row2 * row3 = \delta^{21}(\delta^{52})^{32} + x^8\delta^{80}(x^7\delta^{85})^{32} + x^4\delta^{17}(x^8\delta^{81})^{32} = \delta^{1685} + x\delta^{2800} + x^7\delta^{2609} = \delta^{11} + x\delta^{10} + x^7\delta^5 = (x+x^5+x^7+x^8) + (1+x^3+x^4+x^5+x^6+x^7+x^9+x^{10}) + (1+x+x^3+x^4+x^6+x^8+x^9+x^{10}) = \mathbf{0}$

$row3 * row3 = e + (\delta^{52})^{33} + (x^7\delta^{85})^{33} + (x^8\delta^{81})^{33} = e + \delta^{1716} + \delta^{2805} + \delta^{2673} = e + \delta^{42} + \delta^{15} + \delta^{69} = (x+x^2+x^3+x^4+x^5+x^6+x^7+x^8+x^9+x^{10}) + (x+x^3+x^4+x^5+x^6+x^7+x^8+x^{10}) + (x^3+x^4+x^7+x^8) + (x^2+x^3+x^4+x^7+x^8+x^9) = \mathbf{0}$.

$$\text{gen } C = \begin{pmatrix}
0111111111100000000000000000000000111111111100000110000100111011 00 \\
1011111111100000000000000000000001011111111100000011000010011101 10 \\
1101111111100000000000000000000001101111111100000001100001001110 11 \\
1110111111100000000000000000000001110111111100000000110100100111 01 \\
1111011111100000000000000000000001111011111100000000011110010011 10 \\
1111101111100000000000000000000001111101111100000000001011001001 11 \\
1111110111100000000000000000000001111110111100000000000101100100 11 \\
1111111011100000000000000000000001111111011100000000000110110010 01 \\
1111111101100000000000000000000001111111101100000000000011011001 00 \\
1111111110100000000000000000000001111111110100000000000001110110 01 \\
0000000000001111111111000000000000001111111101111101100111101010 100 \\
0000000000010111111111000000000000001111111111111101100011101010 10 \\
0000000000011011111111000000000000100011111110111111011000111010 101 \\
0000000000011101111111000000000000110001111110011111101110011101 010 \\
0000000000011110111111000000000000111000111111001111110101001110 101 \\
0000000000011111011111000000000000111100011111100111111010100111 010 \\
0000000000011111101111000000000000111110001111110011111101010011 101 \\
0000000000011111110111000000000000111111000111101100111110101001 110 \\
0000000000011111111011000000000000111111100011101100111110101010 011 \\
0000000000011111111101000000000000111111110001110110011110101010 011 \\
0000000000000000000000011111111110000101011011100000010010010110 11 \\
0000000000000000000000010111111110000010101101110000001101001011 01 \\
0000000000000000000000011011111111000001010110111000000110100101 10 \\
0000000000000000000000011101111111100000101001011100000011010010 11 \\
0000000000000000000000011110111111011000001010010111000001101001 01 \\
0000000000000000000000011111011111011000001000010111000011011010 10 \\
0000000000000000000000011111101111011000001000010111000011011010 01 \\
0000000000000000000000011111110111011000001000010111000011011010 00 \\
0000000000000000000000011111111011010110000000000010111010110110 100 \\
0000000000000000000000011111111101010110000000000010111010110110 010 \\
0000000000000000000000011111111110100101011000100000010110010110 11 \\
1111111111100000000000000000000000000000000000000000111111111100000000000 \\
0000000000011111111111000000000000000000000000000000000000000011111111111 \\
0000000000000000000000011111111111111111111111000000000000000000000000
\end{pmatrix}$$

```
IsSelfDual(c);
AutomorphismGroup(c);
PartialWeightDistribution(c,18);
true
Permutation group acting on a set of cardinality 66
Order = 11
    (1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11)(12, 13, 14, 15, 16, 17, 18, 19, 20, 21,
        22)(23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33)(34, 35, 36, 37, 38, 39,
        40, 41, 42, 43, 44)(45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55)(56, 57,
        58, 59, 60, 61, 62, 63, 64, 65, 66)
[ <0, 1>, <12, 1562>, <14, 16566>, <16, 196977>, <18, 1609179> ]
Running Magma V2.20-7.
Seed: 2375350334; Total time: 3.140 seconds; Total memory usage: 219.06MB.
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Binary self-dual $[68, 34, 12]$ codes with new values of the parameter

Table 1: New binary self-dual $[68, 34, 12]$ with $W_{68,2}$, $\gamma = 0$

β	gen mat	$\lvert\mathrm{Aut}(C)\rvert$	$d \le 16, A_d$	β	gen mat	$\lvert\mathrm{Aut}(C)\rvert$	$d \le 16, A_d$
11	$\begin{pmatrix} 01111111111000000000000000000000011111111111010001101111101100010 00 \\ 10111111111000000000000000000000010111111111101000110101110110001 00 \\ 11011111111000000000000000000000011011111111110100011010111011000 00 \\ 11101111111000000000000000000000011101111111011010001101011101100 00 \\ 11110111111000000000000000000000011110111111101110100000101110110 00 \\ 11111011111000000000000000000000011111011111110111010000010111011 00 \\ 11111101111000000000000000000000011111101111110111010000101110100 00 \\ 11111110111000000000000000000000011111110111001101110101100010111 00 \\ 11111111011000000000000000000000011111111011001101110101000101110 00 \\ 11111111101000000000000000000000011111111101100011011101011000101 100 \\ 00000000000011111111110000000000011011100010010110010110000001010 000 \\ 00000000000101111111110000000000011011110001101011001100000000101 000 \\ 00000000000110111111110000000000010110111000110101100100000000010 100 \\ 00000000000111011111110000000000010110111000110101100100000000010 100 \\ 00000000000111101111110000000000010110111000110101100100000000010 100 \\ 00000000000111110111110000000000010001011011001101101101000000000 000 \\ 00000000000111111011110000000000011000101101100101101010100000000 000 \\ 00000000000111111101110000000000011000101101100101101010010100000 000 \\ 00000000000111111110110000000000011000101101100101101000010100000 000 \\ 00000000000111111111010000000000011100010110110010110000010100000 000 \\ 00000000000000000000000000000000011111111111011100010101010010110 100 \\ 00000000000000000000000000000000010111111111011011000110110101001 011 000 \\ 00000000000000000000000000000000011011111111101101110001101010010 011 100 \\ 00000000000000000000000000000000011101111111010110110000110110101 001 100 \\ 00000000000000000000000000000000011110111111001011011010110110101 011 000 \\ 00000000000000000000000000000000011111011111000101101101001101101 010 100 \\ 00000000000000000000000000000000011111101111000101101101001101101 010 100 \\ 00000000000000000000000000000000011111110111000101101101001101101 010 100 \\ 00000000000000000000000000000000011111111011000101101101001101101 010 100 \\ 00000000000000000000000000000000011111111101000101101101001101101 010 100 \\ 00000000000000000000000000000000011111111110100101101101001101101 010 100 \\ 00000000000000000000000000000000011111111111010101101101001101101 010 100 \\ 00000000000000000000000000000000011111111111000000000000000000000 000 \\ 00000000000000000000000000000000011111111111000000000000000000000 000 \\ 00000000000000000000000000000000011111111111000000000000000000000 000 \\ 00000000000000000000000000000000011111111111000000000000000000000 010 \\ 00000000000000000000000000000000011111111111000000000000000000000 001 \end{pmatrix}$	11	$\langle 12, 486\rangle$ $\langle 14, 14872\rangle$ $\langle 16, 174075\rangle$	22	$\begin{pmatrix} 01111111111000000000000000000000011111111111010001101111010001101100 \\ 10111111111000000000000000000000010111111111101000110111010001101100 \\ 11011111111000000000000000000000011011111111110100011011101000110000 \\ 11101111111000000000000000000000011101111111011010001101110100011000 \\ 11110111111000000000000000000000011110111111101110100011011101000110 \\ 11111011111000000000000000000000011111011111110111010000110111010000 \\ 11111101111000000000000000000000011111101111110111010000101110100000 \\ 11111110111000000000000000000000011111110111001101110101000101110100 \\ 11111111011000000000000000000000011111111011001101110101000101110100 \\ 11111111101000000000000000000000011111111101100011011101000110111000 \\ 00000000000011111111110000000000010100011011001101010111000101101100 \\ 00000000000101111111110000000000011010001101100101010100100101101100 \\ 00000000000110111111110000000000011101000110110010101010100010110100 \\ 00000000000111011111110000000000011101000110110010101010100010110100 \\ 00000000000111101111110000000000010111010001101100110100110100101100 \\ 00000000000111110111110000000000011011101000101100110110110100100010 \\ 00000000000111111011110000000000011011101000101011001101101101001000 \\ 00000000000111111101110000000000011011101000101011001101101101000100 \\ 00000000000111111110110000000000010001011011001101101101000101000100 \\ 00000000000111111111010000000000010001011011001101101101000101000100 \\ 00000000000000000000000000000000011111111111010001101101000101000100 \\ 00000000000000000000000000000000010111111111101000110110100010100100 \\ 00000000000000000000000000000000011011111111010110110001101000101100 \\ 00000000000000000000000000000000011101111111001011011010110110101001 \\ 00000000000000000000000000000000011110111111000101101101001101101010 \\ 00000000000000000000000000000000011111011111000101101101001101101010 \\ 00000000000000000000000000000000011111101111000101101101001101101010 \\ 00000000000000000000000000000000011111110111000101101101001101101010 \\ 00000000000000000000000000000000011111111011000101101101001101101010 \\ 00000000000000000000000000000000011111111101000101101101001101101010 \\ 00000000000000000000000000000000011111111110100101101101001101101010 \\ 00000000000000000000000000000000011111111111000000000000000000000000 \\ 00000000000000000000000000000000011111111111000000000000000000000000 \\ 00000000000000000000000000000000011111111111000000000000000000000000 \\ 00000000000000000000000000000000011111111111000000000000000000000010 \\ 00000000000000000000000000000000011111111111000000000000000000000001 \end{pmatrix}$	11	$\langle 12, 530\rangle$ $\langle 14, 14784\rangle$ $\langle 16, 173679\rangle$
33	$\begin{pmatrix} 01111111111000000000000000000000011111111111010001101111010001101100 \\ 10111111111000000000000000000000010111111111101000110111010001101100 \\ 11011111111000000000000000000000011011111111110100011011101000110000 \\ 11101111111000000000000000000000011101111111011010001101110100011000 \\ 11110111111000000000000000000000011110111111101110100011011101000110 \\ 11111011111000000000000000000000011111011111110111010000110111010000 \\ 11111101111000000000000000000000011111101111110111010000101110100000 \\ 11111110111000000000000000000000011111110111001101110101000101110100 \\ 11111111011000000000000000000000011111111011001101110101000101110100 \\ 11111111101000000000000000000000011111111101100011011101000110111000 \\ 00000000000011111111110000000000011000001010001000000010001001100 \\ 00000000000101111111110000000000010011000000010001000010010001000100 \\ 00000000000110111111110000000000010011000000010001000010010001000100 \\ 00000000000111011111110000000000010011000000010001000110010001000 \\ 00000000000111101111110000000000010011000000010001000110010001000 \\ 00000000000111110111110000000000010011000000010001000110010001000 \\ 00000000000111111011110000000000010000010011010000001000010011001 \\ 00000000000111111101110000000000011000001001001000000100001001100100 \\ 00000000000111111110110000000000010000010000010000000110001001100 \\ 00000000000111111111010000000000010000010000010000000110001001100 \\ 00000000000000000000000000000000011111111111010000011101010101001101000000101100 \\ 00000000000000000000000000000000010111111111001000001111010101001101000000010100 \\ 00000000000000000000000000000000011011111111100100000111101010100110100000001000 \\ 00000000000000000000000000000000011101111111110010000011101010110110100000000 \\ 00000000000000000000000000000000011110111111110010000011101010110110100000000 \\ 00000000000000000000000000000000011111011111110010000011101010110110100000000 \\ 00000000000000000000000000000000011111101111110010000011101010110110100000000 \\ 00000000000000000000000000000000011111110111110010000011101010110110100000000 \\ 00000000000000000000000000000000011111111011110010000011101010110110100000000 \\ 00000000000000000000000000000000011111111101110010000011101010110110100000000 \\ 00000000000000000000000000000000011111111110110010000011101010110110100000000 \\ 00000000000000000000000000000000011111111111010010000011101010110110100000000 \\ 00000000000000000000000000000000011111111111000000000000000000000000 \\ 00000000000000000000000000000000011111111111000000000000000000000000 \\ 00000000000000000000000000000000011111111111000000000000000000000000 \\ 00000000000000000000000000000000011111111111000000000000000000000010 \\ 00000000000000000000000000000000011111111111000000000000000000000001 \end{pmatrix}$	11	$\langle 12, 574\rangle$ $\langle 14, 14696\rangle$ $\langle 16, 173283\rangle$	44^*	$\begin{pmatrix} 01111111111000000000000000000000011111111111010001101111010001101000 \\ 10111111111000000000000000000000010111111111101111111111010111000100 \\ 11011111111000000000000000000000011011111111110111111111101011100000 \\ 11101111111000000000000000000000011101111111011010111111101011101100 \\ 11110111111000000000000000000000011110111111101110111111001011101100 \\ 11111011111000000000000000000000011111011111110111010000101110100000 \\ 11111101111000000000000000000000011111101111110111010000101110100000 \\ 11111110111000000000000000000000011111110111001101110101000101110100 \\ 11111111011000000000000000000000011111111011001101110101000101110100 \\ 11111111101000000000000000000000011111111101100011011101000110111000 \\ 00000000000011111111110000000000011111111111010110001001111111110 \\ 00000000000101111111110000000000010111111111101101110001101111111100 \\ 00000000000110111111110000000000010111111111101101110001101111111100 \\ 00000000000111011111110000000000011101111111101011011100110111111100 \\ 00000000000111101111110000000000011101111111101011011100110111111100 \\ 00000000000111110111110000000000011111111111010110111111111101111100 \\ 00000000000111111011110000000000011111111111010110111111111101111100 \\ 00000000000111111101110000000000011111111111010110111111111101111100 \\ 00000000000111111110110000000000011111111111010110111111111101111100 \\ 00000000000111111111010000000000011111111111010110111111111101111100 \\ 00000000000000000000000000000000011111111111011100010011111111110111111111110 \\ 00000000000000000000000000000000010111111111101101110001101111111110111111111100 \\ 00000000000000000000000000000000011011111111101101110001101111111110111111111100 \\ 00000000000000000000000000000000011101111111010110111001101111111110111111111100 \\ 00000000000000000000000000000000011110111111001011011010110111111110111111111100 \\ 00000000000000000000000000000000011111011111000101101101001101111111110111111111100 \\ 00000000000000000000000000000000011111101111000101101101001101111111110111111111100 \\ 00000000000000000000000000000000011111110111000101101101001101111111110111111111100 \\ 00000000000000000000000000000000011111111011000101101101001101111111110111111111100 \\ 00000000000000000000000000000000011111111101000101101101001101111111110111111111100 \\ 00000000000000000000000000000000011111111110100101101101001101111111110111111111100 \\ 00000000000000000000000000000000011111111111000000000000000000000000 \\ 00000000000000000000000000000000011111111111000000000000000000000000 \\ 00000000000000000000000000000000011111111111000000000000000000000000 \\ 00000000000000000000000000000000011111111111000000000000000000000010 \\ 00000000000000000000000000000000011111111111000000000000000000000101 \end{pmatrix}$	110	$\langle 12, 618\rangle$ $\langle 14, 14608\rangle$ $\langle 16, 172887\rangle$

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* Not new code but with larger automorphism group

Table 2: New binary self-dual $[68, 34, 12]$ with $W_{68,2}$, $\gamma = 0$, continued

β	gen mat	$\lvert\mathrm{Aut}\rvert$	$d \le 16, A_d$	β	gen mat	$\lvert\mathrm{Aut}\rvert$	$d \le 16, A_d$
143	0111111111100000000000000000000000000000000000101000110110100111111100 1011111111100000000000000000000000000000000000110100011011010011111100 1101111111100000000000000000000000000000000000111010001101101001111100 1110111111100000000000000000000000000000000000011101000111101001111100 1111011111100000000000000000000000000000000000101110100011111010011100 1111101111100000000000000000000000000000000000011011101000111111001100 1111110111100000000000000000000000000000000000001101110100111111101000 1111111011100000000000000000000000000000000000001101110100111111101000 1111111101100000000000000000000000000000000000001101110100111111101000 1111111110100000000000000000000000000000000000001000110111000111111100 0000000000001111111111000000000000101000110111101110101000011010100 0000000000001011111111100000000000110100011011101110101000001101010 0000000000011011111111000000000000110100011011101110101000001101000 0000000000011101111111100000000000011101000110111011101010000011010 0000000000011110111111000000000000101110100011011110111010000011000 0000000000011111011111000000000000110111010010101110111010100000100 0000000000011111101111000000000000110111010101011110111010000000000 0000000000011111110110000000000000110111011110101111001101010000 0000000000011111111010000000000000100011011100111010111001101000100 0000000000000000000000001111111111001100001101000111111011010100100 0000000000000000000000010111111111000110000111100011111101010100100 0000000000000000000000011011111111000110000111000011111101101010000 0000000000000000000000011101111111000110000111000111101101010100 0000000000000000000000011110111111001100011001111110001100110101000 0000000000000000000000011111011111000110001100011110001101101010100 0000000000000000000000011111101111000110001111111111000101001101100 0000000000000000000000011111110111000110001111111111000101001101100 0000000000000000000000011111111011000110001111111111000101001101100 0000000000000000000000011111111101100011000011111111101010100110100 0000000000000000000000011111111110110000110000111111110101010001100 0000000000011111111110000000000000000000000000000000111111111100 1111111111100000000000000000000000000000000000111111111100000000000000 0000000000000000000000000000000000000000000000111111111110000000000010 0000000000000000000000000000000000000000000000111111111110000000000001	11	$\langle 12, 1014\rangle$ $\langle 14, 13816\rangle$ $\langle 16, 169323\rangle$	154	0111111111100000000000000000000000000000000000101000110110100111111100 1011111111100000000000000000000000000000000000110100011011010011111100 1101111111100000000000000000000000000000000000111010001101101001111100 1110111111100000000000000000000000000000000000011101000111101001111100 1111011111100000000000000000000000000000000000101110100011111010011100 1111101111100000000000000000000000000000000000011011101000111111001100 1111110111100000000000000000000000000000000000001101110100111111101000 1111111011100000000000000000000000000000000000001101110100111111101000 1111111101100000000000000000000000000000000000001101110100111111101000 1111111110100000000000000000000000000000000000001000110111000111111100 0000000000001111111111000000000000101000110111101110101000011010100 0000000000001011111111100000000000110100011011101110101000001101010 0000000000011011111111000000000000110100011011101110101000001101000 0000000000011101111111100000000000011101000110111011101010000011010 0000000000011110111111000000000000101110100011011110111010000011000 0000000000011111011111000000000000110111010010101110111010100000100 0000000000011111101111000000000000110111010101011110111010000000000 0000000000011111110110000000000000110111011110101111001101010000 0000000000011111111010000000000000100011011100111010111001101000100 0000000000000000000000001111111111001100001101000111111011010100100 0000000000000000000000010111111111000110000111100011111101010100100 0000000000000000000000011011111111000110000111000011111101101010000 0000000000000000000000011101111111000110000111000111101101010100 0000000000000000000000011110111111001100011001111110001100110101000 0000000000000000000000011111011111000110001100011110001101101010100 0000000000000000000000011111101111000110001111111111000101001101100 0000000000000000000000011111110111000110001111111111000101001101100 0000000000000000000000011111111011000110001111111111000101001101100 0000000000000000000000011111111101100011000011111111101010100110100 0000000000000000000000011111111110110000110000111111110101010001100 0000000000011111111110000000000000000000000000000000111111111100 1111111111100000000000000000000000000000000000111111111100000000000000 0000000000000000000000000000000000000000000000111111111110000000000010 0000000000000000000000000000000000000000000000111111111110000000000001	11	$\langle 12, 1058\rangle$ $\langle 14, 13728\rangle$ $\langle 16, 168927\rangle$
165	0111111111100000000000000000000000000000000000101000110110100111111100 1011111111100000000000000000000000000000000000110100011011010011111100 1101111111100000000000000000000000000000000000111010001101101001111100 1110111111100000000000000000000000000000000000011101000111101001111100 1111011111100000000000000000000000000000000000101110100011111010011100 1111101111100000000000000000000000000000000000011011101000111111001100 1111110111100000000000000000000000000000000000001101110100111111101000 1111111011100000000000000000000000000000000000001101110100111111101000 1111111101100000000000000000000000000000000000001101110100111111101000 1111111110100000000000000000000000000000000000001000110111000111111100 0000000000001111111111000000000000101000110111101110101000011010100 0000000000001011111111100000000000110100011011101110101000001101010 0000000000011011111111000000000000110100011011101110101000001101000 0000000000011101111111100000000000011101000110111011101010000011010 0000000000011110111111000000000000101110100011011110111010000011000 0000000000011111011111000000000000110111010010101110111010100000100 0000000000011111101111000000000000110111010101011110111010000000000 0000000000011111110110000000000000110111011110101111001101010000 0000000000011111111010000000000000100011011100111010111001101000100 0000000000000000000000001111111111001100001101000111111011010100100 0000000000000000000000010111111111000110000111100011111101010100100 0000000000000000000000011011111111000110000111000011111101101010000 0000000000000000000000011101111111000110000111000111101101010100 0000000000000000000000011110111111001100011001111110001100110101000 0000000000000000000000011111011111000110001100011110001101101010100 0000000000000000000000011111101111000110001111111111000101001101100 0000000000000000000000011111110111000110001111111111000101001101100 0000000000000000000000011111111011000110001111111111000101001101100 0000000000000000000000011111111101100011000011111111101010100110100 0000000000000000000000011111111110110000110000111111110101010001100 0000000000011111111110000000000000000000000000000000111111111100 1111111111100000000000000000000000000000000000111111111100000000000000 0000000000000000000000000000000000000000000000111111111110000000000010 0000000000000000000000000000000000000000000000111111111110000000000001	11	$\langle 12, 1102\rangle$ $\langle 14, 13640\rangle$ $\langle 16, 168531\rangle$	176	0111111111100000000000000000000000000000000000101000110110100111111100 1011111111100000000000000000000000000000000000110100011011010011111100 1101111111100000000000000000000000000000000000111010001101101001111100 1110111111100000000000000000000000000000000000011101000111101001111100 1111011111100000000000000000000000000000000000101110100011111010011100 1111101111100000000000000000000000000000000000011011101000111111001100 1111110111100000000000000000000000000000000000001101110100111111101000 1111111011100000000000000000000000000000000000001101110100111111101000 1111111101100000000000000000000000000000000000001101110100111111101000 1111111110100000000000000000000000000000000000001000110111000111111100 0000000000001111111111000000000000101000110111101110101000011010100 0000000000001011111111100000000000110100011011101110101000001101010 0000000000011011111111000000000000110100011011101110101000001101000 0000000000011101111111100000000000011101000110111011101010000011010 0000000000011110111111000000000000101110100011011110111010000011000 0000000000011111011111000000000000110111010010101110111010100000100 0000000000011111101111000000000000110111010101011110111010000000000 0000000000011111110110000000000000110111011110101111001101010000 0000000000011111111010000000000000100011011100111010111001101000100 0000000000000000000000001111111111001100001101000111111011010100100 0000000000000000000000010111111111000110000111100011111101010100100 0000000000000000000000011011111111000110000111000011111101101010000 0000000000000000000000011101111111000110000111000111101101010100 0000000000000000000000011110111111001100011001111110001100110101000 0000000000000000000000011111011111000110001100011110001101101010100 0000000000000000000000011111101111000110001111111111000101001101100 0000000000000000000000011111110111000110001111111111000101001101100 0000000000000000000000011111111011000110001111111111000101001101100 0000000000000000000000011111111101100011000011111111101010100110100 0000000000000000000000011111111110110000110000111111110101010001100 0000000000011111111110000000000000000000000000000000111111111100 1111111111100000000000000000000000000000000000111111111100000000000000 0000000000000000000000000000000000000000000000111111111110000000000010 0000000000000000000000000000000000000000000000111111111110000000000001	110	$\langle 12, 1146\rangle$ $\langle 14, 13552\rangle$ $\langle 16, 168135\rangle$

Table 3: New binary self-dual $[68, 34, 12]$ with $W_{68,2}$, $\gamma = 0$, continued

β	gen mat	$\lvert\mathrm{Aut}\rvert$	$d \le 16, A_d$	β	gen mat	$\lvert\mathrm{Aut}\rvert$	$d \le 16, A_d$
187	0111111111100000000000000000000000000000000000101000110110100111111100 1011111111100000000000000000000000000000000000110100011011010011111100 1101111111100000000000000000000000000000000000111010001101101001111100 1110111111100000000000000000000000000000000000011101000111101001111100 1111011111100000000000000000000000000000000000101110100011111010011100 1111101111100000000000000000000000000000000000011011101000111101001100 1111110111100000000000000000000000000000000000001101110100111111101000 1111111011100000000000000000000000000000000000001101110100111111101000 1111111011100000000000000000000000000000000000001011101011111111101000 1111111011100000000000000000000000000000000000001011101011111111101000 1111111011100000000000000000000000000000000000001011101011

Table 4: New binary self-dual $[68, 34, 12]$ with $W_{68,2}$, $\gamma = 0$, continued

β	gen mat	|Aut|	$d \leq 16, A_d$	β	gen mat	|Aut|	$d \leq 16, A_d$
231	$\begin{pmatrix} 01111111111000000000000000000000011111111110111111111111101110001000 \\ 10111111111000000000000000000000101111111111011111111111011011000100 \\ 11011111111000000000000000000000110111111111011111111110110111000000 \\ 11101111111000000000000000000000111011111111101111111101011110000 \\ 11110111111000000000000000000000111101111111111011111100101101110000 \\ 11111011111000000000000000000000111110111111111101111110001011011000 \\ 11111101111000000000000000000000111111011111111110111110001011011000 \\ 11111110111000000000000000000000111111101111111111011110001011011000 \\ 11111111011000000000000000000000111111110111111111101111100010110100 \\ 11111111101000000000000000000000111111111011111111110111110001011000 \\ 11111111110100000000000000000000111111111101111111111011011000101100 \\ 00000000000011111111110000000000010100011011011110010101111011111100 \\ 00000000000010111111111000000000001101000110100111100101111101111100 \\ 00000000000011011111111000000000001101000110100111100101111101111100 \\ 00000000000011101111111000000000001101000110100111100111111111011100 \\ 00000000000011110111111000000000001011101000110100111100111111111101100 \\ 00000000000011111011111000000000001011110100001010011111111111110100 \\ 00000000000011111101111000000000001101110100001010011111111111111000 \\ 00000000000011111110111000000000001101110110010100111101111111111100 \\ 00000000000011111111011000000000001101110110010100111101111111111100 \\ 00000000000011111111101000000000001000110111011100101001111011111100 \\ 00000000000000000000000011111111110100111100101110100011111111011100 \\ 00000000000000000000000101111111111010011100101110100011111111101100 \\ 00000000000000000000000110111111111010011110101110100011111111110100 \\ 00000000000000000000000111011111111001010011110110111010011111111000 \\ 00000000000000000000000111101111111001010011110110111010011111111100 \\ 00000000000000000000000111110111111100101001100011011101101111111100 \\ 00000000000000000000000111111011111110010100010001101111101111111100 \\ 00000000000000000000000111111101111111001010001000110111111011111100 \\ 00000000000000000000000111111110110111100101010100011011111011111100 \\ 00000000000000000000000111111111010011110010110100011011111101111100 \\ 00000000000000000000000000000000011111111110000000000001111111111100 \\ 11111111111111111111110000000000000000000000000000000000000000000000 \\ 00000000000000000000000000000000000000000000011111111111000000000010 \\ 00000000000000000000000111111111110000000000000000000000000000000001 \end{pmatrix}$	22	$\langle 12, 1366 \rangle$ $\langle 14, 13112 \rangle$ $\langle 16, 166155 \rangle$	308	$\begin{pmatrix} 01111111111000000000000000000000011111111110100011011010011100100 \\ 10111111111000000000000000000000101111111111010001101101001110000 \\ 11011111111000000000000000000000110111111111101000110010100111000 \\ 11101111111000000000000000000000111011111111101000110010100111100 \\ 11110111111000000000000000000000111101111111101110100011001010011100 \\ 11111011111000000000000000000000111110111111101110100011001010011100 \\ 11111101111000000000000000000000111111011111101110100110010100100 \\ 11111110111000000000000000000000111111101110011011101011110010100000 \\ 11111111011000000000000000000000111111110110001101110001111010100000 \\ 11111111101000000000000000000000111111111011000110111000111100101000 \\ 00000000000011111111110000000000010100011101001101101011110010100 \\ 00000000000010111111111000000000001101000111000011101101111111101111 \\ 00000000000011011111111000000000001101000111000011101101111111101111 \\ 00000000000011101111111000000000001101000111000111101101111111101111 \\ 00000000000011110111111000000000001101101000110100011101111111101111 \\ 00000000000011111011111000000000001101101000110100011111111111101100 \\ 00000000000011111101111000000000001101101000110100011111111111111000 \\ 00000000000011111110111000000000001101101000110100011111111111111000 \\ 00000000000011111111011000000000001101101000110100011011111111111100 \\ 00000000000011111111101000000000001000110111011100101001111011111100 \\ 00000000000000000000000011111111110100011101011101000111101111110100 \\ 00000000000000000000000101111111111010001101011101000110001101110100 \\ 00000000000000000000000110111111111010001111011101000100011011110100 \\ 00000000000000000000000111011111111001010001101101101001010001101100 \\ 00000000000000000000000111101111111001010001000110111011010110000100 \\ 00000000000000000000000111110111111100101000000110111011101000011000 \\ 00000000000000000000000111111011111110010100010001101110011101000100 \\ 00000000000000000000000111111101111111001010001000110111011101000100 \\ 00000000000000000000000111111110110111100101010100011011111011101000 \\ 00000000000000000000000111111111010011110010110100011011111011101000 \\ 00000000000000000000000000000000011111111110000000000000000000000000 \\ 11111111111111111111110000000000000000000000000000000000000000000000 \\ 00000000000000000000000000000000000000000000011111111111000000000010 \\ 00000000000011111111111000000000000000000000000000000000000000000001 \end{pmatrix}$	44	$\langle 12, 1674 \rangle$ $\langle 14, 12496 \rangle$ $\langle 16, 163383 \rangle$
330	$\begin{pmatrix} 01111111111000000000000000000000011111111110100011011010011100100 \\ 10111111111000000000000000000000101111111111010001101101001110000 \\ 11011111111000000000000000000000110111111111101000110010100111000 \\ 11101111111000000000000000000000111011111111101000110010100111100 \\ 11110111111000000000000000000000111101111111101110100011001010011100 \\ 11111011111000000000000000000000111110111111101110100011001010011100 \\ 11111101111000000000000000000000111111011111101110100110010100100 \\ 11111110111000000000000000000000111111101110011011101011110010100000 \\ 11111111011000000000000000000000111111110110001101110001111010100000 \\ 11111111101000000000000000000000111111111011000110111000111100101000 \\ 00000000000011111111110000000000010100011101001101101011110010100 \\ 00000000000010111111111000000000001101000111000011101101111111101111 \\ 00000000000011011111111000000000001101000111000011101101111111101111 \\ 00000000000011101111111000000000001101000111000111101101111111101111 \\ 00000000000011110111111000000000001101101000110100011101111111101111 \\ 00000000000011111011111000000000001101101000110100011111111111101100 \\ 00000000000011111101111000000000001101101000110100011111111111111000 \\ 00000000000011111110111000000000001101101000110100011111111111111000 \\ 00000000000011111111011000000000001101101000110100011011111111111100 \\ 00000000000011111111101000000000001000110111011100101001111011111100 \\ 00000000000000000000000011111111110100011101011101000111101111110100 \\ 00000000000000000000000101111111111010001101011101000110001101110100 \\ 00000000000000000000000110111111111010001111011101000100011011110100 \\ 00000000000000000000000111011111111001010001101101101001010001101100 \\ 00000000000000000000000111101111111001010001000110111011010110000100 \\ 00000000000000000000000111110111111100101000000110111011101000011000 \\ 00000000000000000000000111111011111110010100010001101110011101000100 \\ 00000000000000000000000111111101111111001010001000110111011101000100 \\ 00000000000000000000000111111110110111100101010100011011111011101000 \\ 00000000000000000000000111111111010011110010110100011011111011101000 \\ 00000000000000000000000000000000011111111110000000000000000000000000 \\ 11111111111111111111110000000000000000000000000000000000000000000000 \\ 00000000000000000000000000000000000000000000011111111111000000000010 \\ 00000000000011111111111000000000000000000000000000000000000000000001 \end{pmatrix}$	22	$\langle 12, 1762 \rangle$ $\langle 14, 12320 \rangle$ $\langle 16, 162591 \rangle$				

Table 5: New binary self-dual $[68, 34, 12]$ with $W_{68,1}$

β	gen mat	|Aut|	$d \le 16, A_d$	β	gen mat	|Aut|	$d \le 16, A_d$
115	0111111111100000000000000000000000011111111111010001101110100011011001 1011111111100000000000000000000000010111111111101000110111010001101100 1101111111100000000000000000000000011011111111110100011011101000110001 1110111111100000000000000000000000011101111111110110100011011101000110 1111011111100000000000000000000000011110111111110111010001101110100011 1111101111100000000000000000000000011111011111110111010001101110100011 1111110111100000000000000000000000011111101111110111010001101110100011 1111111011100000000000000000000000011111110111110110110001101110100011 1111111101100000000000000000000000011111111011110110110001101110100011 1111111110100000000000000000000000011111111101110110110001101110100011 0000000000001111111110000000000000101000110111010001101110100011011000 0000000000001011111111100000000000110100011010101000111110000000000100 0000000000011011111111100000000000110100011010101000111110000000000100 0000000000011101111111100000000000111010001101010100011110000000000100 0000000000011110111111100000000000111010001110101000111011000000000000 0000000000011111011111100000000000101110100011110101000100110000000000 0000000000011111101111100000000000101110100011110101000100110000000000 0000000000011111110111100000000000110111010001110101000000110000000000 0000000000011111111011100000000000110111010001110101000000011000000000 0000000000011111111101100000000000110111010001110101000000001100000000 0000000000011111111110100000000000110111010001110101000000000110000000 0000000000000000000000001111111111010011100111001001101000101011111100 0000000000000000000000001011111111101001110011100100110100010101111100 0000000000000000000000001101111111101001111001110010011110001010100 0000000000000000000000001110111111100101001111011100100111100010100 0000000000000000000000001111011111110010100111101110010011110001010100 0000000000000000000000001111101111110010100110111011100100111100010100 0000000000000000000000001111110111110010100110111011100100111100010100 0000000000000000000000001111111011110010100110111011100100111100010100 0000000000000000000000001111111101110010100110111011100100111100010100 0000000000000000000000001111111110110010100110111011100100111100010100 0000000000000000000000001111111111010010100110111011100100111100010100 0000000000000000000000001111111111101010010110111011100100111100010100 1111111111100000000000111111111110000000000000000000000000000000000011 0000000000001111111111000000000000000000000000111111111111000000000000 0000000000000000000000001111111111111111111111111111111111000000000001 0000000000000000000000001111111111111111111111111111111111111111111110	11	$\langle 12, 902\rangle$ $\langle 14, 9944\rangle$ $\langle 16, 219483\rangle$	247	0111111111100000000000000000000000000000000010100011101101001111111100 1011111111100000000000000000000000000000000011010001101101001111111100 1101111111100000000000000000000000000000000011101000110110100111111100 1110111111100000000000000000000000000000000011101000111110100111111100 1111011111100000000000000000000000000000000010111101000111110100111100 1111101111100000000000000000000000000000000011101101000111110100111100 1111110111100000000000000000000000000000000011101101000111110100111100 1111111011100000000000000000000000000000000011101101000111110100111100 1111111101100000000000000000000000000000000011101101000111110100111100 1111111110100000000000000000000000000000000011101101000111110100111100 0000000000001111111110000000000000101000110111010001101110100011011000 0000000000001011111111100000000000110100011010101000111110000000000100 0000000000011011111111100000000000110100011010101000111110000000000100 0000000000011101111111100000000000111010001101010100011110000000000100 0000000000011110111111100000000000111010001110101000111011000000000000 0000000000011111011111100000000000101110100011110101000100110000000000 0000000000011111101111100000000000101110100011110101000100110000000000 0000000000011111110111100000000000110111010001110101000000110000000000 0000000000011111111011100000000000110111010001110101000000011000000000 0000000000011111111101100000000000110111010001110101000000001100000000 0000000000011111111110100000000000110111010001110101000000000110000000 0000000000000000000000001111111111010011100111001001101000101011111100 0000000000000000000000001011111111101001110011100100110100010101111100 0000000000000000000000001101111111101001111001110010011110001010100 0000000000000000000000001110111111100101001111011100100111100010100 0000000000000000000000001111011111110010100111101110010011110001010100 0000000000000000000000001111101111110010100110111011100100111100010100 0000000000000000000000001111110111110010100110111011100100111100010100 0000000000000000000000001111111011110010100110111011100100111100010100 0000000000000000000000001111111101110010100110111011100100111100010100 0000000000000000000000001111111110110010100110111011100100111100010100 0000000000000000000000001111111111010010100110111011100100111100010100 0000000000000000000000001111111111101010010110111011100100111100010100 1111111111100000000000111111111110000000000000000000000000000000000011 0000000000001111111111000000000000000000000000111111111111000000000000 0000000000000000000000001111111111111111111111111111111111000000000001 0000000000000000000000001111111111111111111111111111111111111111111110	11	$\langle 12, 1430\rangle$ $\langle 14, 8888\rangle$ $\langle 16, 214731\rangle$
280	0111111111100000000000000000000000000000000010100011101101001111111100 1011111111100000000000000000000000000000000011010001101101001111111100 1101111111100000000000000000000000000000000011101000110110100111111100 1110111111100000000000000000000000000000000011101000111110100111111100 1111011111100000000000000000000000000000000010111101000111110100111100 1111101111100000000000000000000000000000000011101101000111110100111100 1111110111100000000000000000000000000000000011101101000111110100111100 1111111011100000000000000000000000000000000011101101000111110100111100 1111111101100000000000000000000000000000000011101101000111110100111100 1111111110100000000000000000000000000000000011101101000111110100111100 0000000000001111111110000000000000101000110111010001101110100011011000 0000000000001011111111100000000000110100011010101000111110000000000100 0000000000011011111111100000000000110100011010101000111110000000000100 0000000000011101111111100000000000111010001101010100011110000000000100 0000000000011110111111100000000000111010001110101000111011000000000000 0000000000011111011111100000000000101110100011110101000100110000000000 0000000000011111101111100000000000101110100011110101000100110000000000 0000000000011111110111100000000000110111010001110101000000110000000000 0000000000011111111011100000000000110111010001110101000000011000000000 0000000000011111111101100000000000110111010001110101000000001100000000 0000000000011111111110100000000000110111010001110101000000000110000000 0000000000000000000000001111111111010011100111001001101000101011111100 0000000000000000000000001011111111101001110011100100110100010101111100 0000000000000000000000001101111111101001111001110010011110001010100 0000000000000000000000001110111111100101001111011100100111100010100 0000000000000000000000001111011111110010100111101110010011110001010100 0000000000000000000000001111101111110010100110111011100100111100010100 0000000000000000000000001111110111110010100110111011100100111100010100 0000000000000000000000001111111011110010100110111011100100111100010100 0000000000000000000000001111111101110010100110111011100100111100010100 0000000000000000000000001111111110110010100110111011100100111100010100 0000000000000000000000001111111111010010100110111011100100111100010100 0000000000000000000000001111111111101010010110111011100100111100010100 1111111111100000000000111111111110000000000000000000000000000000000011 0000000000001111111111000000000000000000000000111111111111000000000000 0000000000000000000000001111111111111111111111111111111111000000000001 0000000000000000000000001111111111111111111111111111111111111111111110	11	$\langle 12, 1562\rangle$ $\langle 14, 8624\rangle$ $\langle 16, 213543\rangle$	291	0111111111100000000000000000000000000000000010100011101101001111111100 1011111111100000000000000000000000000000000011010001101101001111111100 1101111111100000000000000000000000000000000011101000110110100111111100 1110111111100000000000000000000000000000000011101000111110100111111100 1111011111100000000000000000000000000000000010111101000111110100111100 1111101111100000000000000000000000000000000011101101000111110100111100 1111110111100000000000000000000000000000000011101101000111110100111100 1111111011100000000000000000000000000000000011101101000111110100111100 1111111101100000000000000000000000000000000011101101000111110100111100 1111111110100000000000000000000000000000000011101101000111110100111100 0000000000001111111110000000000000101000110111010001101110100011011000 0000000000001011111111100000000000110100011010101000111110000000000100 0000000000011011111111100000000000110100011010101000111110000000000100 0000000000011101111111100000000000111010001101010100011110000000000100 0000000000011110111111100000000000111010001110101000111011000000000000 0000000000011111011111100000000000101110100011110101000100110000000000 0000000000011111101111100000000000101110100011110101000100110000000000 0000000000011111110111100000000000110111010001110101000000110000000000 0000000000011111111011100000000000110111010001110101000000011000000000 0000000000011111111101100000000000110111010001110101000000001100000000 0000000000011111111110100000000000110111010001110101000000000110000000 0000000000000000000000001111111111010011100111001001101000101011111100 0000000000000000000000001011111111101001110011100100110100010101111100 0000000000000000000000001101111111101001111001110010011110001010100 0000000000000000000000001110111111100101001111011100100111100010100 0000000000000000000000001111011111110010100111101110010011110001010100 0000000000000000000000001111101111110010100110111011100100111100010100 0000000000000000000000001111110111110010100110111011100100111100010100 0000000000000000000000001111111011110010100110111011100100111100010100 0000000000000000000000001111111101110010100110111011100100111100010100 0000000000000000000000001111111110110010100110111011100100111100010100 0000000000000000000000001111111111010010100110111011100100111100010100 0000000000000000000000001111111111101010010110111011100100111100010100 1111111111100000000000111111111110000000000000000000000000000000000011 0000000000001111111111000000000000000000000000111111111111000000000000 0000000000000000000000001111111111111111111111111111111111000000000001 0000000000000000000000001111111111111111111111111111111111111111111110	11	$\langle 12, 1606\rangle$ $\langle 14, 8536\rangle$ $\langle 16, 213147\rangle$

Table 6: New binary self-dual $[68, 34, 12]$ with $W_{68,1}$, continued

β	gen mat	$\lvert\mathrm{Aut}\rvert$	$d \le 16, A_d$	β	gen mat	$\lvert\mathrm{Aut}\rvert$	$d \le 16, A_d$
313	$\begin{pmatrix} 01111111111000000000000000000000011111111111010001101101001111 00100 \\ 10111111111000000000000000000000101111111111101000110110100111 00000 \\ 11011111111000000000000000000000110111111111110100011001010011 11000 \\ 11101111111000000000000000000000111011111111101100011001010011 11100 \\ 11110111111000000000000000000000111101111111011010001100101001 11100 \\ 11111011111000000000000000000000111110111111011101000110010100 11100 \\ 11111101111000000000000000000000111111011110110110111010011100 10100 \\ 11111110111000000000000000000000111111101110011011101011110010 10000 \\ 11111111011000000000000000000000111111110110001101110011110010 10100 \\ 11111111101000000000000000000000111111111011000110111000111001 00000 \\ 00000000000011111111110000000000000101001011011110100011001000001 00 \\ 00000000000010111111111000000000000010100101101111010001100100000 100 \\ 00000000000011011111111000000000000001010010110111101000110000000 100 \\ 00000000000011101111111000000000000001000101001011110100111001000 000 \\ 00000000000011110111111000000000000010100101000010111100001110010 000 \\ 00000000000011111011110000000000000010100010101000101111000011100 100 \\ 00000000000011111101110000000000000010100010101000101111000011100 100 \\ 00000000000011111110110000000000000010010100011101000101110000011 100 \\ 00000000000011111111010000000000000010100010101000010111000001110 000 \\ 00000000000000000000000011111111110010111000011100110010001110111 100 \\ 00000000000000000000000101111111110001011000011100110011001110111 100 \\ 00000000000000000000000110111111110000101110010111001100111011011 100 \\ 00000000000000000000000111011111110000010111001011100110111001101 100 \\ 00000000000000000000000111101111110000000101110010111001111100111 000 \\ 00000000000000000000000111110111111000000101110010111001111100111 000 \\ 00000000000000000000000111111011111100000010011100101110101111001 100 \\ 00000000000000000000000111111101101110000000100110010111101111100 100 \\ 00000000000000000000000111111110111100000000100110010111101111110 000 \\ 11111111111000000000000111111111110000000000000000000000000000000 011 \\ 00000000000111111111110000000000000111111111110000000000000000000 011 \\ 00000000000000000000000111111111111111111111111111111110000000000 001 \\ 00000000000000000000000111111111111111111111111111111110000000000 1110	11	$\langle 12, 1694\rangle$ $\langle 14, 8360\rangle$ $\langle 16, 212355\rangle$	324	$\begin{pmatrix} 01111111111000000000000000000000011111111111010001101110100011 01100 \\ 10111111111000000000000000000000101111111111101000110111010001 10100 \\ 11011111111000000000000000000000110111111111110100011011101000 11000 \\ 11101111111000000000000000000000111011111111101100011011101000 11100 \\ 11110111111000000000000000000000111101111111011101000110111010 00100 \\ 11111011111000000000000000000000111110111111011101000110111010 00000 \\ 11111101111000000000000000000000111111011110110110111010001101 10000 \\ 11111110111000000000000000000000111111101110011011101000110111 01000 \\ 11111111011000000000000000000000111111110110001101110100011011 10100 \\ 11111111101000000000000000000000111111111011000110111010001101 11000 \\ 00000000000011111111110000000000000101001011011110100011000011 11000 \\ 00000000000010111111111000000000000010100101101111010001100000 11100 \\ 00000000000011011111111000000000000001010010110111101000110000 11100 \\ 00000000000011101111111000000000000001000101001011110100111000 00000 \\ 00000000000011110111111000000000000010100101000010111100001110 00000 \\ 00000000000011111011110000000000000010100010101000101111000011 10100 \\ 00000000000011111101110000000000000010100010101000101111000011 10100 \\ 00000000000011111110110000000000000010010100011101000101110000 11100 \\ 00000000000011111111010000000000000010100010101000010111000001 11000 \\ 00000000000000000000000011111111110010111000011100110010001110 11100 \\ 00000000000000000000000101111111110001011000011100110011001110 11100 \\ 00000000000000000000000110111111110000101110010111001100111011 01100 \\ 00000000000000000000000111011111110000010111001011100110111001 10100 \\ 00000000000000000000000111101111110000000101110010111001111100 11100 \\ 00000000000000000000000111110111111000000101110010111001111100 11100 \\ 00000000000000000000000111111011111100000010011100101110101111 00100 \\ 00000000000000000000000111111101101110000000100110010111101111 10000 \\ 00000000000000000000000111111110111100000000100110010111101111 11000 \\ 11111111111000000000000111111111110000000000000000000000000000000 011 \\ 00000000000111111111110000000000000111111111110000000000000000000 011 \\ 00000000000000000000000111111111111111111111111111111110000000000 1101 \\ 00000000000000000000000111111111111111111111111111111110000000000 1110	11	$\langle 12, 1738\rangle$ $\langle 14, 8272\rangle$ $\langle 16, 211959\rangle$
379	$\begin{pmatrix} 01111111111000000000000000000000011111111111010001101111101100 01000 \\ 10111111111000000000000000000000101111111111101000110101110110 00100 \\ 11011111111000000000000000000000110111111111110100011010111011 01000 00 \\ 11101111111000000000000000000000111011111111101100011010111011 0100 \\ 11110111111000000000000000000000111101111111011101000100101110 11000 \\ 11111011111000000000000000000000111110111111011101000000101110 1100 \\ 11111101111000000000000000000000111111011110110110110100010111 0100 \\ 11111110111000000000000000000000111111101110011011101011000101 11000 \\ 11111111011000000000000000000000111111110110001101110101100010 11100 \\ 11111111101000000000000000000000111111111011000110111010110001 01100 \\ 00000000000011111111110000000000000100100110011111100001010001 11000 00 \\ 00000000000010111111111000000000000010010011011111100000100001 1110000 \\ 00000000000011011111111000000000000010010010010101111100000010001 11100 \\ 00000000000011101111111000000000000001000100100001011111000000100011 100 \\ 00000000000011110111111000000000000001100010010000101111100000100011 100 \\ 00000000000011111011110000000000000001100010010000101111110000100 100 \\ 00000000000011111101110000000000000010011000100100001011111110000100000 \\ 00000000000011111110110000000000000010011000101100001011101110000100 \\ 00000000000011111111010000000000000010011000111100001011001100001000 \\ 00000000000011111111010000000000000010010011000111000010100011100001 00 \\ 00000000000000000000000011111111110001100100100110001111101001111110 \\ 00000000000000000000000110111111110001100100100110001101100111111100 \\ 00000000000000000000000111011111110001000110010110011100011011001111 00 \\ 00000000000000000000000111101111110010001100111001110001101100111100 \\ 00000000000000000000000111110111110100100011000111001110111101100110 00 \\ 00000000000000000000000111111011110100100011000111001110111101100100 \\ 00000000000000000000000111111101111100100011000111001110111101100100 \\ 00000000000000000000000111111110111100100100011000111001011111101100 \\ 00000000000000000000000111111111011100100100011000111001011111101100 \\ 11111111111111111111111000000000000000000000000000000000000000000 011 \\ 00000000000000000000000111111111111111111111111111111110000000000 1 \\ 00000000000111111111110000000000000111111111110000000000000000000 1110	22	$\langle 12, 1958\rangle$ $\langle 14, 7832\rangle$ $\langle 16, 209979\rangle$				

Binary self-dual $[70, 35, 12]$ code with new values of the parameter

Code with $W_{70,1}$ for $\beta = 842$, $|\mathrm{Aut}(C)| = 11$ with generator matrix

$$G = \begin{pmatrix}
0111111111100000000000000000000000011111111111010001101101001111001000 0\\
1011111111100000000000000000000000101111111111101000110110100111100000 0\\
1101111111100000000000000000000000110111111111110100011001010011110000 0\\
1110111111100000000000000000000000111011111111011101000110010100111100 00\\
1111011111100000000000000000000000111101111111011101000110010100111000 0\\
1111101111100000000000000000000000111110111111101110100011001010011000 0\\
1111110111100000000000000000000000111111011110110111010011001010010000 0\\
1111111011100000000000000000000000111111101110011011101011100101000000 0\\
1111111101000000000000000000000000111111110110001101110101110010100000 0\\
1111111110100000000000000000000000111111111011000110111000111001010000 0\\
0000000000001111111111000000000000101000111010011101101011101111110000 0\\
0000000000010111111111000000000000110100011100001110110111111011110000 0\\
0000000000011011111111000000000000110100011110001110110111111011110000 0\\
0000000000011101111111000000000000101101000110100011101111111101110000 0\\
0000000000011110111111000000000000101101000110100011101111111110110000 0\\
0000000000011111011111000000000000111011010001101000111011111111010000 0\\
0000000000011111101111000000000000111011010001101000111111111111100000 0\\
0000000000011111110111000000000000111011010101101000110111111111110000 0\\
0000000000011111111011000000000000011101101110110100011011111111110000 0\\
0000000000011111111101000000000000100011101101101101000110111111110000 0\\
0000000000000000000000111111111101000111010110100011000110111010000 0\\
0000000000000000000000101111111111010001101011101000110001101110000 0\\
0000000000000000000000110111111111010100011111011101000010001101110000 0\\
0000000000000000000000111011111111011010001101101110100101000110110000 0\\
0000000000000000000000111101111111101101000100110111101011010001101000 0\\
0000000000000000000000111110111111110110100000011011110111010001100000 0\\
0000000000000000000000111111011110111011011010010001101110011101000110000 0\\
0000000000000000000000111111101110011101101001000110111011101000100000 0\\
0000000000000000000000111111110110001101101101000110111011101000000000 0\\
0000000000000000000000111111111011000110110110100011010110111010000000 0\\
1111111111100000000000000000000000000000000000000000000000000000000111\\
0000000000011111111110000000000000000000000000000000000000000000001000\\
0000000000000000000000111111111100000000000000000000000000111111111110110\\
0000000000000000000000000000000011111111110000000000011111111110101\\
0000000000000000000000000000000000000000000011111111111111111111110011
\end{pmatrix}$$

```
IsSelfDual(c);
AutomorphismGroup(c);
PartialWeightDistribution(c,18);
true
Permutation group acting on a set of cardinality 70
Order = 11
    (1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11)(12, 13, 14, 15, 16, 17, 18, 19, 20, 21,
        22)(23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33)(34, 35, 36, 37, 38, 39,
        40, 41, 42, 43, 44)(45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55)(56, 57,
        58, 59, 60, 61, 62, 63, 64, 65, 66)
[ <0, 1>, <12, 1684>, <14, 7998>, <16, 154539>, <18, 1229316> ]

Running Magma V2.20-7.
Seed: 3638495913; Total time: 3.720 seconds; Total memory usage: 141.44MB.
```

gen $C_\varphi = \begin{pmatrix} e & 0 & 0 & e & \delta & \delta^{63} \\ 0 & e & 0 & \delta^{62} & x^7\delta^{62} & x^4 e \\ 0 & 0 & e & \delta^{62} & x^3\delta & x^8\delta \end{pmatrix}$. Orthogonality check for C_φ with Hermitian inner product $(u, v) = \sum_{i=1}^{6} u_i v_i^{32}$:

$row1 * row1 = e + e + \delta^{33} + (\delta^{63})^{33} = e + \delta^{33} + \delta^{2079} = \delta^{33} + \delta^{33} = \mathbf{0}$

$row2 * row2 = e + (\delta^{62})^{33} + (x^7\delta^{62})^{33} + (x^4 e)^{33} = e + e + e + e = \mathbf{0}$

$row2 * row1 = \delta 62 + x^7\delta^{94} + x^4(\delta^{63})^{32} = \delta^{62} + x^7\delta + x^4\delta^{63} = (1 + x^2 + x^6 + x^7 + x^8 + x^{10}) + (x^2 + x^3 + x^5 + x^6 + x^7 + x^9) + (1 + x^3 + x^5 + x^8 + x^9 + x^{10}) = \mathbf{0}$

$row3 * row1 = \delta^{62} + x^3\delta^{33} + x^8\delta\delta^{63.32} = \delta^{62} + x^3\delta^{33} + x^8\delta^{64} = (1 + x^2 + x^6 + x^7 + x^8 + x^{10}) + (1 + x + x^2 + x^4 + x^5 + x^6) + (x + x^4 + x^5 + x^7 + x^8 + x^{10}) = \mathbf{0}$

$row2 * row3 = \delta^{62.33} + x^7\delta^{62}(x^3\delta)^{32} + x^4(x^8\delta)^{32} = e + x^4\delta + x^7\delta^{32} = (x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^8 + x^9 + x^10) + (1 + x^2 + x^3 + x^4 + x^6 + x^10) + (1 + x + x^5 + x^7 + x^8 + x^9) = \mathbf{0}$

$row3 * row3 = e + (\delta^{62})^{33} + (x^3\delta)^{33} + (x^8\delta)^{33} = e + e + \delta^{33} + \delta^{33} = \mathbf{0}$.